P system Final State Probabilites*

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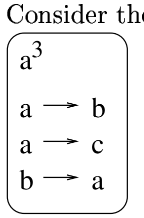
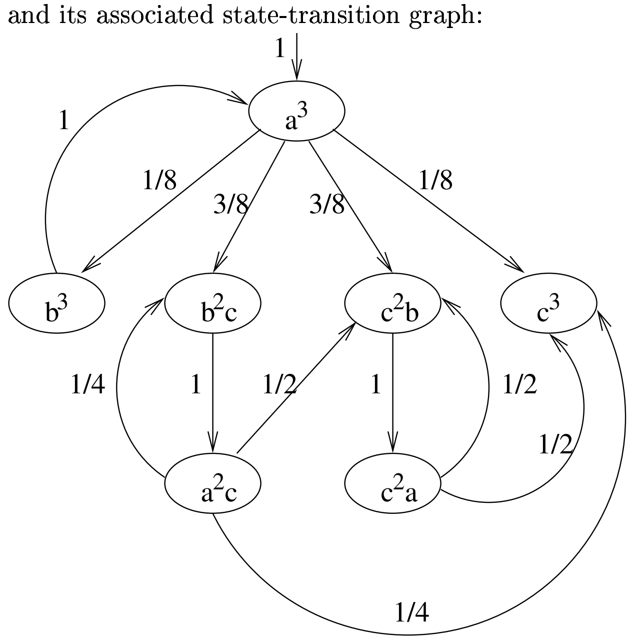
In order to compute the probabilities of the final states of a P system, we regard the system as a discrete first-order Markov process, that is, a process such that its state at a certain step depends only on the previous state (is independent of all other previous states). The states of the process are the configurations of the P system.

Consider the following P system,



and its associated state-transition graph:



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The graph nodes represent the P system configurations and the edges the possible transitions. We specify along each edge the probability of its associated transition.

The considered P system is not trivial for our purposes, since it is non-deterministic and contains simple and connected cycles, and its associated graph is not a tree. We note that even if the graph would not contain cycles, it could still be more general than a tree, because a certain state could be reacheable by different paths (ultimately, a node in the graph could have more than one parent).

We note that there is a single final state, the one described by the configuration (multiset) c^3. Evidently, the probability of the system reaching this configuration must be 1.

Since we start with certainty from a^3, we place an edge with associated probability 1 coming from outside with target a^3.

The essential procedure for solving the problem is the cycle-handling procedure, that is, how do we treat cycles? How do we interpret, for instance, that a^3 is maximally parallel rewritten to b^3 ($a^3 \Rightarrow b^3$) with a probability of $\frac{1}{8}$, and then $b^3 \Rightarrow a^3$ with probability 1? What happens is that, if we are in a^3, there is a $\frac{1}{8}$ probability of returning to this state after two *mpr*s. From this configuration the system can evolve with the represented probabilites in the following configurations, b^3, b^2c, c^2b, c^3. Thus, there is an additional probability of $\frac{1}{8} \cdot \frac{1}{8}$ of reaching b^3again, and by consequence of reaching a^3again, and so on.

To compute the final state probabilities, we need to account for these repeated passes through configurations due to cycles in the graph. If we were to draw the graph in a tree-like manner, ignoring the fact that some configurations are repeated, we could work with simple probabilities. But since we consider the cycle visualization essential, we use the concept of *passes through* a certain configuration. It is easy to see that all evolutions pass through a^3 one time, since it is the initial state, then 1 in 8 evolutions will reach a^3 a second time, and 1 in 64 will reach it a third time and so on. So, the total possible passes through a^3 are given by $1 + \frac{1}{8} + \frac{1}{64} + \cdots$.

$P_t(a^3) = 1 + \sum_{n=1}^{\infty} \frac{1}{8^n}$

For any state, its *passes through* number can be computed as: $P_t = P_0(1 + P_c)$, that is, the probability of reaching it the first time, P_0, added with $P_0 \cdot P_c$, the additional *passes through* allowed by cycles. For a^3, this is given by:

$$P_t(a^3) = 1(1 + \sum_{n=1}^{\infty} \frac{1}{8^n}) = 1 + \frac{1}{7} = \frac{8}{7}$$

$$\begin{aligned} P_t(b^2c) &= P_0(b^2c)(1 + P_c(b^2c)) = \frac{3}{8} P_t(a^3)(1 + P_c(b^2c)) \\ &= \frac{3}{8} \cdot \frac{8}{7}(1 + \sum_{n=1}^{\infty} \frac{1}{4^n}) = \frac{3}{7}(1 + \frac{1}{3}) = \frac{3}{7} \cdot \frac{4}{3} = \frac{4}{7} \end{aligned}$$

$$P_t(a^2c) = P_t(b^2c) = \frac{4}{7}$$

$$
\begin{aligned}
P_t(c^2b) &= (\frac{3}{8}P_t(a^3) + \frac{1}{2}P_t(a^2c))(1 + P_c(c^2b)) = \\
&= (\frac{3}{7} + \frac{2}{7})(1 + \sum_{n=1}^{\infty} \frac{1}{2^n}) = \frac{10}{7}
\end{aligned}
$$

$$
P_t(c^2a) = P_t(c^2b) = \frac{10}{7}
$$

For the final states, P_c is obviously 0, so its *passes through* number is equal to its probability, $P_t = P_0$.

The only final state in this case is c^3:

$$
\begin{aligned}
P_0(c^3) = P_t(c^3) &= \frac{1}{8}P_t(a^3) + \frac{1}{4}P_t(a^2c) + \frac{1}{2}P_t(ac^2) = \\
&= \frac{1}{8} \cdot \frac{8}{7} + \frac{1}{4} \cdot \frac{4}{7} + \frac{1}{2} \cdot \frac{10}{7} = 1
\end{aligned}
$$

This shows that the sum of the final state probabilities is 1, q.e.d.