P systems behavioral analysis

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For a transitional P system w/o active membranes, we have defined:

Definition 1 *OR-bigraph*

The Objects-Rules bigraph (ORg) *is the bigraph* $ORg = (O + R, E)$ *where* O *is the set of objects and* R *is the set of rules of a P system, and* $E = \{(o, r) | o \in O, r \in R\}$.

Definition 2 *OR-matrix*

The OR-matrix (OR) *is the adjacency matrix of the OR-bigraph* (ORg).

The coupling definitions:

Definition 3 *the Direct Coupling of two rules*

We have three types of objects in a transition rule: Input (I), Output (O) and Promoters/Inhibitors (PI) objects. We consider all direct interaction situations in descending priority order:

Table 1: All direct interaction situations for two rules

r1	r2	DCf	Comment
I	I	0	the rules sharing an input object influence each other in the same step
I	PI	0	the same holds if one rule has as input the promoter/inhibitor of the other
I	O	1	if one rule has a input the output of the other or the output of one is the promoter/inhibitor of the other, the rules need another step to influence each other
O	PI	1	
O	O	∞	even if they have a common output or common promoter/inhibitor the rules do not influence each other
PI	PI	∞	
other	other	∞	the rules have no common objects so they do not influence each other

The DCf *represents the number of steps required for two rules to directly influence each other (based just on their objects).*

Algorithm 1 Shortest-Path Matrix of RC (Floyd's algorithm)

$SP = RC$ {the values 0, 1 are represented in the DC matrix}
for $k = 0$ to $N - 1$ **do**
  for $i = 0$ to $N - 1$ **do**
    for $j = 0$ to $N - 1$ **do**
      $SP_{i,j} = min(SP_{i,j}, SP_{i,k} + SP_{k,j})$
    end for
  end for
end for
$RC_f = SP + 1$ {RC_f is the final Rule Cost(Correlation) matrix}

Definition 4 *Direct Coupling matrix*

The Direct Coupling matrix (DC) *is defined like* $DC_{i,j} = DCf(r_i, r_j)$.

Definition 5 *RR-graph*

The Rules-Rules graph is given its $RRg = (R, E)$ *where* R *is the set of rules of the* P *system, and* $E = \{(r_1, r_2) | r_1, r_2 \in R \wedge DCf(r_1, r_2) \text{ is finite}\}$.

Definition 6 *RR-matrix*

The RR-matrix (RR) *is the adjacency matrix of the RR-graph* RRg.

Definition 7 *Rules Cost matrix*

If $X = OR^T \cdot OR$,

the Rule Cost matrix (RC) *is calculated in the following way*

$$RC_{i,j} = \begin{cases} 0, & X_{i,j} = 0 \\ DC_{i,j}. & else \end{cases}$$

We compute the shortest-path matrix of the RR-graph using Floyd's algorithm:

Definition 8 *P System temperature*

$T_{Ps} = \frac{\|RC_f\|_F}{n} - 1$ *where* $\|A\|_F = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} a_{i,j}^2}$ *is Frobenius norm,* n *is the number of* P *system rules(the rows/cols from* RC_f*).*

Definition 9 *hotter system*

It is named that a P system is hotter than another if he have a greater temperature.

"Hot media are, therefore, low in participation, and cool media are high in participation or completion by the audience." M.McLuhan *Understanding Media:Media Hot and Cold*, by Routledgeand Kegan Paul, UK 1964.

Theorem 1 *Consider a transitional P system without active membranes; if an object is eliminated from any rule the resulting P system will be* ***hotter*** *than the original.*

Proof: to be done (trivial from the direct coupling of two rules definition)
□

Definition 10 *correlated systems*

A P system is named correlated system if the final rule correlation matrix RC_f has only finite elements.

Theorem 1 *If $P(n) = \frac{n^2(n^2-1)}{12}$ is the 4-dimensional pyramidal numbers, then the maximum temperature of an corelated system is*

$$T_{max} = \frac{\sqrt{2 \cdot P(n+1) - n}}{n} - 1$$

Proof: *to be done.*
□

Theorem 1 *Consider a transitional P system without active membranes; if all the rules influenced each other in the first step then the temperature of this P system would be $T = 0$, where n is the number of the rules.*

Proof: In a P system with all the rules influence each other in step 0, $DC_{i,j} = 0$ [1], that means the matrix OR doesn't have any rule or column completely '0', from here and RC-matrix definition will obtain that $X_{i,j} \neq 0, \forall i, j \leq n$, that involve $RC_{i,j} = DC_{i,j}, \forall i, j \leq n$ [2].

[1],[2] and Floyd's algorithm involve that $SP_{i,j} = RC_{i,j} = 0, \forall i, j \leq n$ and inn the final of algorithm $RC_{fi,j} = 1, \forall i, j \leq n$ [3].

Applied the temperature definition for [3] is obtaining $T = 0$,q.e.d.
□

Definition 11 *the nondeterministic correlation*

If two rules from a membrane with the same priority have the same input object or an input object of one is the promoter/inhibitor of the other then they are nondeterministically correlated. OR is posible to say there exist a nondeterminist between those rules.