SYNASC09 - Symbolic Computation

m-Hilbert Polynomial and Arbitrariness of the General Solution of Partial Differential Equations

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Abstract—Using the framework of formal theory of partial differential equations, we consider a method of computation of the m-Hilbert polynomial (i.e. Hilbert polynomial with multivariable), which generalizes the Seiler's theorem of Hilbert polynomial with single variable. Next we present an approach to compute the number of arbitrary functions of positive differential order in the general solution, and give a formally well-posed initial problem. Finally, as applications the Maxwell equations and weakly overdetermined equations are considered.

Superfluous S-polynomials in Strategy-Independent Gr¨obner Bases

Grant Olney Passmore and Leonardo de Moura

Abstract—Using the machinery of proof orders originally introduced by Bachmair and Dershowitz in the context of canonical equational proofs, we give an abstract, strategyindependent presentation of Gr¨obner basis procedures and prove the correctness of two classical criteria for recognising superfluous S-polynomials, Buchberger's criteria 1 and 2, w.r.t. arbitrary fair and correct basis construction strategies. To do so, we develop a general method for proving the strategy-independent correctness of superfluous Spolynomial criteria which seems to be quite powerful. We also derive a new superfluous S-polynomial criterion which is a generalization of Buchberger-1 and is proved to be correct strategy-independently.

A Hybrid Algorithm for Solving 7 Parameters Transformation

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Abstract—The 7 parameters transformation is a well-known problem in engineering sciences such as Computer Vision, Survey Engineering and Photogrammetry. To solve this problem, we use an algebraic solver in which we need to transform the input system into an equivalent system, but better adapted such as a Gröbner basis. For operational applications of this problem, we need to compute this basis as fast as possible. In this paper, we describe an efficient hybrid algorithm which employs both Buchberger algorithm and Faugèr's F5 algorithm to compute this basis. Using this algorithm, we find a trace for computing the Gröbner basis of the polynomial ideal generated by the equations of the system. This trace can provide directly the Gröbner basis of the ideal generated by the system if the coordinates of input points change without useless computations.

Toom-Cook 8-way For Long Integers Multiplication

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Abstract—Toom-Cook algorithms are subquadratic polynomial multiplication methods that are efficiently used also for long integers. Generally speaking, only the degree 2 (Karatsuba), 3 and 4 version are used in practice. In this paper we analyze a high (8-way – degree 7) version, showing that it can be effective for long integers whose digits number lies in a certain range. Comparison with GMP 4.3.0 library shows that the gain can be quite significant, both for multiplication and squaring.

Computing Self-intersection Loci of Parametrized Surfaces Using Regular Systems and Grobner Bases

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Abstract—The computation of self-intersection loci of parametrized surfaces is needed for constructing trimmed parametrizations and describing the topology of the considered surfaces in real settings. This paper presents two general and effective methods for determining self-intersection loci of rationally parametrized surfaces. One of the methods, based on regular systems, is capable of computing the exact parametric locus of self-intersection of a given surface and the other, based on Gr¨obner bases, can compute the minimal variety passing through the exact parametric locus. The relation between the results computed by the two methods is established and two algorithms for computing parametric loci of self-intersection are described. Experimental results and comparisons with some existing methods show that our algorithms have a good performance for parametrized surfaces.

Lagrange Interpolation Method for Implicitizing Parametric Surfaces

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Abstract - An efficient algorithm for finding the implicit equation of a parametric surface defined by its parametric equations is presented. The algorithm is based on the efficient computation of the Dixon's matrix and Lagrange interpolation. Main features of our approach are the facts that it can not only reduces the unnecessary computations but also considerably reduces the problem of intermediate expression. Thus we can speed up the italicization of properly parametric surfaces and consider problems of not small size.

Point Location in arrangements of algebraic hypersurfaces — informal presentation

Rafael Grimson

Abstract—We prove that for each particular family of polynomials G ={ g1, ..., gs}, it is possible to design an algorithm that determines the face of the partition of R^n consisting of the connected components of the sign conditions of the family G, containing a query point whose search time grows only logarithmically in s. The computational model considered is that of algebraic computation trees. Our general algorithm is based on a divide-and-conquer strategy applied to the data structure obtained form Collins' [5] cylindrical algebraic decomposition. This divide-and-conquer strategy is, on its turn, based on Thom's Lemma.

On Computer Algebra Aid to Analytic Integration within Tetragonal Finite Element

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Abstract—The paper presents computer algebra aided evaluation of integrals of the finite element stiffness matrix. There was shown how the user can interactively carry out computations leading to the result as simple as possible. The obtained results make it possible to improve the finite element accuracy and reliability. Moreover the procedures may become quicker.