SYMBOLIC ANALYSIS BY DECOMPOSITION OF COMPLEX ANALOG CIRCUITS

Lucia Dumitriu, Mihai Iordache, Ilie Luican

"Politehnica" University of Bucharest, Electrical Engineering Department,
Spl. Independentei 313, Cod 77 206, Bucharest ROMANIA,
Phone/Fax (+401) 411 11 90 dlucia@hertz.pub.ro

Abstract. In this paper we present some new tearing techniques to systematically formulate the state equations in symbolic normal-form for linear and/or nonlinear time-invariant large-scale analog circuits. The excess elements of the first and of the second kind are unitarily treated in order to allow a symbolic representation of the circuit with a minimum number of state variables. The symbolic analysis by state variable via diakoptic approach of opamp μA 741 is presented, in a case when the symbolic analysis of the whole circuit is not possible.
Key words: – tearing method, symbolic analysis, excess elements, state equations, state matrix, and eigenvalues.

I. INTRODUCTION

The growth in complexity of IC chips has generated the need for more efficient simulation techniques, which radically differ from the standard simulation techniques [1,2,3,4].

The decomposition techniques used in large-scale analog circuits have the following advantages:

1. The development of parallel processing systems for solving several subcircuits simultaneously leads to a reduction in computation time;
2. Solving only the non-latent subcircuits sequentially in the proper order using the latency principle yields saving in computation time and memory space.

In the last years some new algorithms and computer programs have been developed in this area in order to be used in CAD of analog integrated circuits. Design problems as noise analysis, sensitivity computation, distortion analysis of weakly nonlinear circuits, symbolic pole/zero extraction, circuit sizing based on optimization of analytic models, can be efficiently solved by means of symbolic methods [3,4]. The hierarchical analysis using graph decomposition [5,6], matrix decomposition [7], or splitting of the circuits containing nullors [8,9], leads to significant reduction of CPU time.

In this paper we present three tearing techniques based on *splitting of some central nodes* from the normal tree, that allow the circuit state equations' formulation, and which can be applied both for linear and/or nonlinear large-scale analog circuits.

II. TEARING TECHNIQUES FOR LAGE-SCALE CIRCUIT ANALYSIS

In order to formulate the circuit state equations, a normal tree of the circuit is selected. This is a special tree which contains certain circuit elements in the following priority order: all independent and controlled voltage sources, all nonlinear voltage-controlled resistors, as many capacitors as possible, as many controlling branches of the current-controlled voltage sources and of the current-controlled current sources as possible (these branches are considered as resistive branches having null resistances), as many resistors as possible, and as few inductors as possible. It does not contain any independent and/or controlled current source, and any nonlinear current-controlled resistor. The capacitors that are not included in the normal tree are called *excess capacitors*, and the inductors that are included in the normal tree are called *excess inductors*.

The nodes of the normal tree in which at least two tree branches and as much as links are connected, are called *central nodes*, and those in which only one tree branch is connected are called *external nodes*.

In the following we present a decomposition procedure of a large-scale circuit by splitting of some central nodes from its normal tree. All tearing methods require that the elements in a subcircuit must be strongly interconnected, whereas the different subcircuits must be weakly interconnected [2,15,16,30] each other. We have to choose p central nodes, which are considered as *reference nodes*. These p reference nodes are torn apart by p *splitting cut-sets* (SCS) into $p + 1$ subcircuits. In all tearing variants these p+1 subcircuits can be interconnected either by ideal independent voltage sources (e_{con}) and ideal independent current sources (j_{con}) (independent connection sources), or by VCVS's with unity voltage gain and CCCS's with unity current gain (controlled connection sources).

The connection sources are defined in respect to the reference node of each splitting cut-set.

Remarks:

1. The assignment of the connection sources to the subcircuits has to keep, as much as possible, the structure of the state variable vector of the whole circuit.

2. Since a subcircuit contains the same modeling primitives as the original circuit (that will be restored finally by reconnecting all the subcircuits), the accuracy of the simulation results is not affected.

When independent connection sources are used, the currents and the voltages of the ideal independent voltage sources and of the ideal independent current sources must satisfy, at each splitting cut-set Σ_k, $k=\overline{1,p}$, the following relations:

$$\begin{aligned} i_{e_{con},k} &= j_{con,k}, \\ e_{con,k} &= v_{j_{con},k} \end{aligned}. \tag{1}$$

In this case the subcircuits are separately processed, and finally, by aggregation of the state equations of all subcircuits (that suppose the elimination of the connection sources) the whole circuit solution is obtained. For the large-scale integrated circuits' analysis, the parallel processing can be used.

If controlled connection sources are used, they must satisfy the controlling equations:

$$\begin{aligned} e_{con,k} &= 1 \cdot v_{j_{con},k}, \\ j_{con,k} &= 1 \cdot i_{e_{con},k} \end{aligned}, \tag{2}$$

we have to solve the complete state equation system from the beginning, but the equations are much better structured.

III. SETTING UP THE SYMBOLIC STATE EQUATIONS

After the generation of a circuit normal tree, the essential incidence matrix (*EIM*) $\boldsymbol{D}_m$, associated to the normal tree in the subcircuit S_m, is generated.

Writing the Kirchhoff's laws for tree currents, respectively for link voltages, in the subcircuit S_m, we obtain:

KCL: $$\boldsymbol{i}_{t,m} = -\boldsymbol{D}_m \boldsymbol{i}_{l,m} \tag{3}$$

KVL: $$\boldsymbol{v}_{l,m} = \boldsymbol{D}_m^{\mathrm{t}} \boldsymbol{v}_{t,m}. \tag{4}$$

The constitutive equations of the linear and nonlinear circuit elements, and the definition equations of the controlled sources must be added to these equations. We assume that the controlling variables of the controlled sources can be expressed in terms of some resistor voltages, resistor currents, or state variables.

The number of state-variables associated with the circuit being equal to the number of dynamic elements minus the number of excess elements, we choose as state variables the tree capacitor voltages $\boldsymbol{v}_{Ct,m}$ and the link inductor currents $\boldsymbol{i}_{Ll,m}$. The complete equation system must be solved in respect of these variables.

Solving this system, we obtain the symbolic state equation in the normal form

$$\dot{\boldsymbol{x}}_m = \boldsymbol{A}_m \boldsymbol{x}_m + \boldsymbol{B}_m \boldsymbol{y}_m + \boldsymbol{B}_{1,m} \dot{\boldsymbol{y}}_m \tag{5}$$

where the matrices $\boldsymbol{A}_m$, $\boldsymbol{B}_m$ and $\boldsymbol{B}_{1,m}$ have the elements in symbolic form, and

$$\boldsymbol{x}_m = \begin{bmatrix} \boldsymbol{v}_{Ct,m}^{\mathrm{t}} & \boldsymbol{i}_{Ll,m}^{\mathrm{t}} \end{bmatrix}^{\mathrm{t}}, \qquad \boldsymbol{y}_m = \begin{bmatrix} \boldsymbol{v}_{e_i,m}^{\mathrm{t}} & \boldsymbol{v}_{e_{con},m}^{\mathrm{t}} & \boldsymbol{i}_{j_i,m}^{\mathrm{t}} & \boldsymbol{i}_{j_{con},m}^{\mathrm{t}} \end{bmatrix}^{\mathrm{t}}. \tag{6}$$

The vectors $\boldsymbol{v}_{e_{con},m}$ and $\boldsymbol{i}_{j_{con},m}$, $m=\overline{1,p+1}$, must be expressed in respect of all state variables, of the voltages of all independent voltage sources, and of the currents of all independent current sources. For that, we use the Kirchhoff's current law corresponding to the cut-sets attached to the connection voltage sources, and the Kirchhoff's voltage law corresponding to the loops attached to the connection current sources, obtaining for each subcircuit S_m, $m=\overline{1,p+1}$:

$$\begin{aligned} \boldsymbol{i}_{e_{con},m} = &-\boldsymbol{D}_{e_{con}C,m} \boldsymbol{i}_{Cl,m} - \boldsymbol{D}_{e_{con}R,m} \boldsymbol{i}_{Rl,m} - \boldsymbol{D}_{e_{con}L,m} \boldsymbol{i}_{Ll,m} - \\ &- \boldsymbol{D}_{e_{con}j_c,m} \boldsymbol{i}_{j_c,m} - \boldsymbol{D}_{e_{con}j_i,m} \boldsymbol{i}_{j_i,m} - \boldsymbol{D}_{e_{con}j_{con},m} \boldsymbol{i}_{j_{con},m} \end{aligned} \tag{7}$$

$$\boldsymbol{v}_{j_{con},m} = \boldsymbol{D}^{t}_{e_i j_{con},m}\boldsymbol{v}_{e_i,m} + \boldsymbol{D}^{t}_{e_c j_{con},m}\boldsymbol{v}_{e_c,m} + \boldsymbol{D}^{t}_{e_{con} j_{con},m}\boldsymbol{v}_{e_{con},m} + \\ + \boldsymbol{D}^{t}_{Cj_{con},m}\boldsymbol{v}_{Ct,m} + \boldsymbol{D}^{t}_{Rtj_{con},m}\boldsymbol{v}_{Rt,m} + \boldsymbol{D}^{t}_{Ltj_{con},m}\boldsymbol{v}_{Lt,m}, \tag{8}$$

Because the two connection sources ($e_{con,k}$, $j_{con,k}$) attached to a splitting cut-set Σ_k are assigned to different subcircuits (S_{m-1} and S_m), they must satisfy the following relations:

$$\boldsymbol{i}_{e_{con},m-1} = \boldsymbol{i}_{j_{con},m} \tag{9}$$

$$\boldsymbol{v}_{j_{con},m} = \boldsymbol{v}_{e_{con},m-1}. \tag{10}$$

By the aggregation of the equations (5) for all subcircuits taking into account the equations (7)-(10), we obtain the state equation in symbolic form for the whole circuit:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{y} + \boldsymbol{B}_1\dot{\boldsymbol{y}} \tag{11}$$

where the matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{B}_1$ have the elements in symbolic form, and

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{v}^{t}_{Ct} & \boldsymbol{i}^{t}_{Ll} \end{bmatrix}^{t}, \qquad \boldsymbol{y} = \begin{bmatrix} \boldsymbol{v}^{t}_{e_i} & \boldsymbol{i}^{t}_{j_i} \end{bmatrix}^{t}. \tag{12}$$

The algorithm for large-scale circuit decomposition, and the method to systematically formulate the state equations in symbolic normal-form for linear and/or nonlinear time-invariant large-scale analog circuits with excess elements, was implemented in **SYSEG** – **SY**mbolic **S**tate **E**quation **G**eneration-program. Starting from the circuit netlist, the program performs the decomposition of the large-scale analog circuit into p + 1 subcircuits, identifying the p splitting cut-sets, and formulates the state equations in symbolic form. It uses the algorithm to formulate in symbolic normal form the state equations of the circuit, but it avoids the matrices' multiplication. Kirchhoff's laws are simple written by successively generation of the cut-sets and of the loops respectively, that makes the program very efficient regarding the computing time and memory.

IV. EXAMPLE

When the symbolic analysis of the whole large-scale circuit is not possible, the tearing method becomes the unique alternative and this is its main advantage.

A good example is μA 741 operational amplifier (Fig. 1), whose partially symbolic state equations in normal form we want to formulate, in order to extract the circuit state matrix. If the transistors are modeled with six parameters, the small-signal equivalent circuit of the amplifier in open-loop configuration contains 26 nodes and 140 primitive elements.

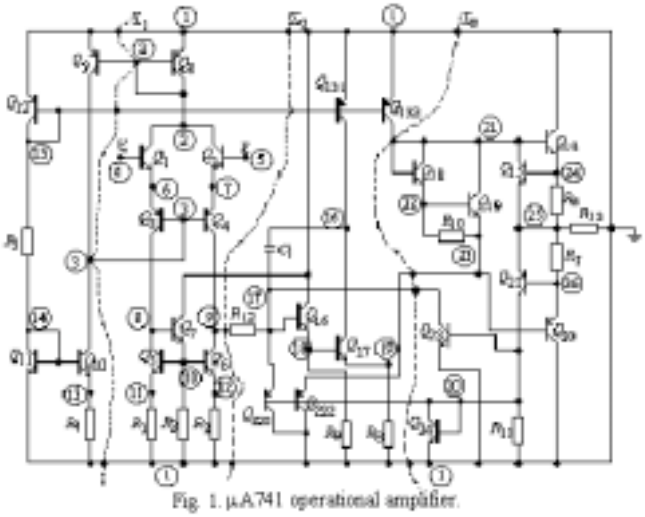

Fig. 1. μA741 operational amplifier.

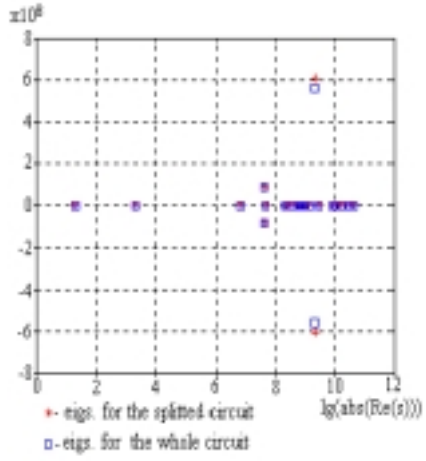

Fig.2. Pole location

Applying our computing program we can obtain the symbolic state equations in normal form for different number of symbols and for different locations of the corresponding circuit elements. In some situations, though, the symbolic expressions being too large, the symbolic manipulator fails. This happens, for example, when we take as symbols the parameters of the transistors Q_4, Q_{131}, Q_{16}, Q_{17}, Q_{21},

and R_8, R_9, R_{11}, C_1. To solve the problem we have performed a diakoptic analysis tearing the circuit by Σ_1, Σ_2, and Σ_3 cut-sets, in four subcircuits. These were independently analyzed, and the partially symbolic state equations of the entire circuit, with 36 symbols (including the six connection resistances, and the complex frequency *s*), were obtained by aggregation. The numerical values of the eigenvalues obtained by the numeric analysis of the whole circuit are in a good agreement with those obtained by symbolic diakoptic analysis (Fig. 2). The maximum error is 5.14% for a complex conjugate eigenvalue that can be seen in the figure.

V. CONCLUSION

In order to extend the topological method of the normal tree [12,13] to generate in symbolic form the state equations of the large-scale analog circuits with excess elements, we have developed several tearing techniques, based on splitting of some central nodes (called reference nodes) from the normal tree.

In respect with the reference nodes, the connection (independent or controlled) sources are introduced. Voltage and current connection sources keep the relation between the subcircuits, and they must be introduced avoiding the generation of *C-E* loops or *L-J* cutsets. When this is not possible, the currents of e_{con} and the voltages of j_{con} depend also on the derivatives of the state variables of the subcircuits that make the subcircuit aggregation more difficult.

To avoid this situation, some small (big) resistances must be inserted in series (in parallel) with each e_{con} (j_{con}) belonging to a *C-E* loop (*L-J* cutset). These connection resistors simplify the computation of the currents (voltages) of e_{con} (j_{con}) in respect with the state variables and input variables of the corresponding subcircuits. In this way, the aggregation process is directly performed, and a bigger number of symbols can be manipulated, with a very good accuracy of the results.

The choice of the values of these resitances depends on the circuit structure and on the values of the resitances contained in the initial circuit.

The connection resistors introduce new eigenvalues that have the magnitudes much bigger than the biggest magnitude of the circuit eigenvalues, and that do not affect the dynamic behavior of the circuit.

The main advantage of the diakoptic approach is that it can be used in the cases when the symbolic analysis of the whole circuit is not possible.

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